Sheet D: Series Expansion, analytical tool for stability, limiting models.

Maclaurin Series:

$$f(x) \approx \sum_{n=0}^{\infty} \left.\frac{d^n f}{dx^n}\right|_{x=0} \frac{x^n}{n!}$$

Taylor Series:

$$f(x) \approx \sum_{n=0}^{\infty} \left.\frac{d^n f}{dx^n}\right|_{x=a} \frac{(x-a)^n}{n!}$$

a) i) $y = e^{2t} \approx 1 + 2t + \frac{4t^2}{2!}$

$\approx 1 + 2t + 2t^2 \ldots$

ii) $y = \frac{1}{1+t}$ | $\frac{dy}{dt} = \frac{-1}{(1+t)^2}$ $\quad \frac{d^2y}{dt^2} = \frac{2}{(1+t)^3}$

$y \approx 1 - x + x^2 \ldots$

iii) $y = \frac{1}{1+t^2}$ | $\underbrace{\frac{dy}{dt} = \frac{-2t}{(1+t^2)^2}}_{\text{will be } 0}$ $\quad \frac{d^2y}{dt^2} = \frac{8t^2}{(1+t^2)^3} - \frac{2}{(1+t^2)^2}$

$$\underbrace{\frac{d^3y}{dt^3} = \frac{24t}{(1+t^2)^3} - \frac{48t^3}{(1+t^2)^4}}_{\text{will be } 0}$$

$$\frac{d^4y}{dt^4} = -\frac{288}{(t^2+1)^4} + \frac{24}{(t^2+1)^3} + \frac{384t^4}{(t^2+1)^5}$$

$$\therefore \; y \approx \underline{1} - t^2 + t^4 \; \ldots$$

b) $y = e^2$