

Introduction to Probability

Question Sheet 2

*(Questions on this sheet correspond to Lectures 4&5. Questions marked with * require knowledge of basic calculus.)*

Question 1

On the average, a certain spectrophotometer part lasts 10 years. The length of time the part lasts is exponentially distributed.

- a) What is the probability that a spectrophotometer part lasts more than 7 years?
- b) On the average, how long would 5 computer parts last if they are used one after another?

Question 2

A drug is produced in a chamber with temperature X , where X is a random variable that follows a normal distribution with $\mu = 21^\circ\text{C}$ and $\sigma = 5^\circ\text{C}$. If $X < 25^\circ\text{C}$, the production cost per unit is £20, whereas if $X \geq 25^\circ\text{C}$ the production cost per unit is £30. Find the expected production cost per unit for this drug.

Question 3

A box contains 3 green and 5 red balls. If we pick balls and then replace them, find:

- a) The probability that the third time we pick a ball, we get a red ball for the second time.
- b) The probability that the 4th time we pick a ball, a red ball appears for the first time
- c) The expected number of times we pick a ball until we get a red ball for the second time.

Question 4

For a group of 100 students, find:

- a) The expected number of days of the year that are birthdays of exactly 3 people.
- b) The expected number of distinct birthdays

***Question 5**

A continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{4x(9-x^2)}{81} & \text{if } 0 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

- a. Verify that $f(x)$ is a valid probability density function
- b. Find (a) the mean, (b) the mode, (c) the median.

***Question 6**

a. Let X be a continuous random variable with probability density function:

$$f(x) = \begin{cases} 4x^3 & \text{if } 0 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $P(X \leq \frac{2}{3} | X > \frac{1}{3})$

b. Let X be continuous random variable with probability density function:

$$f(x) = \begin{cases} x^2 \left(2x + \frac{3}{2}\right) & \text{if } 0 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

If $Y = \frac{2}{X} + 3$, find $\text{Var}(Y)$.

Question 7

A polar bear is hunting for food. From its current position, there are three different routes. The first one brings it back to the same point after 1 hours of walking. The second route brings it back to the same point after 6 hours of walking. The last road leads to the food source after 2 hours of walking. Assuming that the bear chooses a road equally likely at all times, what is the mean time until the bear gets some food?

Numerical Answers

Question 1

- a. $P(X \geq 7) = 0.4966$
- b. 50 years

Question 2

£22.12

Question 3

- a. 0.293
- b. 0.033
- c. 3.2

Question 4

- a. 0.93
- b. 87.6

Question 5

- i. 1.6
- ii. $\sqrt{3}$
- iii. ~ 1.62

Question 6

- a. 0.1875
- b. ~ 1.97

Question 7

$E(T) = 9$

