Introduction to Probability
Question Sheet 1

(Questions 1-4 correspond to Lectures 1&2 and Questions 4-8 correspond to Lecture 3)

Question 1

In a natural habitat, grey wolves, foxes and coyotes co-exist. The current population is estimated to be 60 foxes, 30 coyotes and 10 wolves. If each animal is equally likely to be sighted first, find the probability of:

   a. coyote sighted first
   b. wolf sighted first
   c. fox sighted second if either coyote or wolf sighted first and removed from the habitat so they can be fitted with a tracking device.

Question 2

A transmitter emits signal "0" or "1" in the ratio 1:2. A receiver R_1 is incorrect in 2% of the cases and receiver R_2 is incorrect in 3% of the cases. Find:

   a. the probability that R_1 reports that the transmitted signal is "0"
   b. the probability that R_2 reports that the transmitted signal is "1"
   c. if R_1 reports signal "0" and R_2 reports signal "1", which one of the two receivers is most likely to be correct?

Question 3

   a. Of a batch of 22 imported oak saplings, 12 are healthy and 10 have hidden oak processionary moth eggs. A tree health officer is aware that there may be an issue and chooses 5 trees at random to inspect in detail. As there are slightly more healthy trees than infested trees, we might predict that the batch will consist of 3 healthy trees and 2 infested trees. How many ways are there to select 3 healthy trees and 2 infested trees from the 22 saplings?
   b. What is the probability that a sample of five trees, chosen at random from the batch of 22 imported trees, consists of three healthy and two infested trees?

c. Suppose instead that all 5 of the trees selected are healthy (and so the infestation remains undetected). How many ways are there that the sample could have been chosen at random from the batch to give this result?

d. What is the probability the sample of five, chosen at random from the batch consists only of healthy trees?

Question 4

A spam filter on a mail server scrutinises messages for the appearance of key words that make it likely that the message is spam.

The filter works as follows:

- it is assumed that evidence for spam is that the subject message of the email contains the sentence "check this out".

According to some historical data we know that:

- 40% of emails are spam
- 1% of spam emails have "check this out" in the subject line
- 0.4% of non-spam emails have "check this out" in the subject line.

Find the probability that an email is spam given that it has the phrase "check this out" in the subject.

Question 5

The number of particles emitted from a radioactive substance follows a Poisson distribution with parameter $\lambda=4$ particles per minute. Find the probability that:

a. three particles are emitted in one minute

b. at least one particle is emitted in a minute

c. three particles are emitted in 2 minutes

Question 6

a. The survival rate of a specific disease is forty percent. In a certain day eight patients are admitted into hospital with that disease. What are the chances that three will survive?

b. A new drug is discovered that can cure the disease but can only be administered in a way that has a success rate of reaching the affected area of

0.6. There usually are four doses of the drug administered. Find the chances of:

i.   three doses successfully administered
ii. no doses successfully administered
iii. at least one dose successfully administered
iv. at most two doses successfully administered

Question 7

A scientist randomly selects dolphins until she finds one that has been tagged with a tracking device, so that she can collect the data on the pod's movements. The probability that she finds the dolphin with the tracking device is 0.2.

a. What is the probability that the scientist must select exactly 4 dolphins before she finds one with a tracking device?
b. What is the probability that she must select more than 6 dolphins before she finds one with a tracking device?

Question 8

The number of results we get per minute from a simple computational model is Poisson random variable with mean 3.

a. Find the probability that no results come in a given 1 minute period.
b. Find the probability that at least two results will come in a given two minute period.

Numerical answers

Question 1
  a. 0.3
  b. 0.1
  c. 20/33

Question 2
  a. $P(K_0) = 0.34$
  b. $P(L_1) = 0.657$
  c. R_2

Question 3
  a. 9900
  b. 0.3759
  c. 792
  d. 0.03

Question 4
  P= 5/8 = 0.625

Question 5
  a. $P(X = 3) = 0.195$
  b. $P(X \geq 1) = 0.98$
  c. $P(X = 3) = 0.0286$

Question 6
  a. 0.279
  b.
     i.   P(Three Doses) = 0.3456
     ii. P(No Dose) = 0.0256
     iii. P(At least one) = 0.9744
     iv. P(At most two) = 0.5248

Question 7
  a. P(X=4) = 0.1024
  b. P(X>6) = 0.262

Question 8
  a. $P(X = 0) = e^{-3}$
  b. $1 - 7e^{-6}$